<DCSE> DEPARTMENT OF COMPUTER SCIENCE AND ENGINEERING

Generalized Mirror Symmetry of a Point Set in E3

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Abstract

This work deals with generalized reflectional symmetry of point sets in E3. The generalization is based on replacing a plane as a mirroring surface by a general curved surface. The goal of this work was to formulate a definition of such a generalization of mirror symmetry and to propose methods capable of finding a set of points lying on the surface of symmetry or the surface itself. The functionality of the proposed algorithms was tested on various types of input data.

Introduction and Problem Formulation

The aim of this work was to formalize the problem of reflecting points over a curved surface and to subsequently detect this surface, which we call the "surface of symmetry". The ability to detect the surface of symmetry can provide a means for compressing, describing, or deforming 3D objects. The work was part of the international project GAČR: 21-08009K Generalized Symmetries and Equivalences of Geometric Data.

Reflecting Points over a Curved Surface

Two approaches were considered for reflecting points over a curved surface. They are both based on the known properties of conventional reflectional symmetry. When the reflecting surface is a plane, points and their images are at the same distance from the plane of symmetry and their connecting lines are perpendicular to this plane. Moreover, the connecting lines of all pairs of points are parallel to each other. When replacing the plane with a curved surface, these conditions cannot generally be met at once. Therefore, the mirroring direction of the points was defined as either fixed, i.e., in such a manner that all connecting lines are parallel to one another, or in such a way that the connecting lines of each point and its image are perpendicular to the surface of symmetry.

When considering perpendicular mirroring, the task is non-trivial. If we know the surface to be detected, it is necessary to find the normal to this surface from a point that does not lie on it. In the context of this work, one algorithm was presented that predetermines the specific surface of symmetry to be searched for. A sphere was chosen as this surface because the normals of a sphere always pass through its centre.

Proposed Algorithms

As part of the work, four algorithms were proposed for the detection of the surface of symmetry in a set of points. All but one focused on the detection of points lying on the surface of symmetry. The exception is the method searching for a sphere as the surface of symmetry.

The first algorithm considers a fixed direction of mapping and separates the input set of points with a suitable plane into two subsets to find the surface of symmetry. The normal vector of the separating plane is considered the direction of mapping. The suitability of mapping is determined by the distance between the image and the nearest other point in the input set, and by the distance between the detected points lying on the surface of symmetry and the input set. Preference is given to small distance of images and large distance from the input set.

The second algorithm complements this approach and takes into account the normals of the input points, which were approximated using multiple linear regression. This method favors pairs of points whose normals make a small angle. This and the following algorithms consider perpendicular direction of mapping.

The third approach works only with the use of approximate normals and searches for images of points in the directions determined by the normals of points.

The last of the proposed algorithms modifies the existing method of planar symmetry detection. The modification consists of replacing the plane with a sphere.

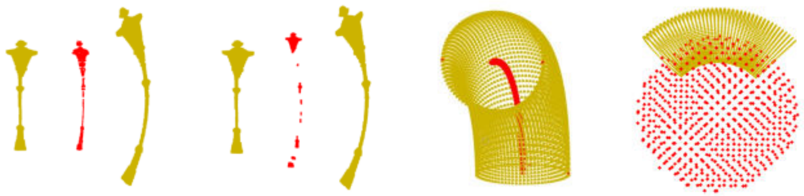
Results

The proposed algorithms were tested on various types of input data. It was found that algorithms based on the separation of the input set do not provide ideal results if the given set is not clearly separable by a plane. For the first of the algorithms, it is also preferable that the point density is almost uniform throughout the input set.

A disadvantage of the algorithm that considers only approximated normals of points is that it assumes the same length of all connecting lines of pairs of points. It is not suitable for point sets that do not represent a sampled surface, as it strongly depends on approximated normals of points.

The use of the algorithm searching for a spherical surface requires the specification of the range of radii to be tested. The results of this method were usually satisfactory if the range of radii was selected appropriately.

Examples of results obtained by the proposed algorithms are shown below and they appear in the order they were described:



Surfaces of symmetry (red) detected in different objects using different algorithms

Conclusion and Future Work

A generalization of mirror symmetry was defined and algorithms able to detect points lying on the surface of symmetry have been proposed. The results show that each of the presented generalized symmetry detection methods is suitable for a different type of input data.

This thesis served as an introductory study in the context of a specific generalization of symmetry, and it is expected that further research will follow and build upon this work.

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